Enrollment No: _____ Exam Seat No:_____

C.U.SHAH UNIVERSITY
Winter Examination-2018

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT2 **Branch: Diploma (All)**

Semester : 2 **Date : 23/10/2018** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:
(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) $\lim_{\theta\to 0}\frac{\sin 5\theta}{\theta}$ = _____

(A) 1 (B) 0 (C) 1/5 (D) 5

b) $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$ = _____

(A) e^2 (B) e (C) $e^{1/2}$ (D) None of these

c) $\lim_{x\to 0}\frac{x^2+x+1}{x+1}$ = _____

(A) 1 (B) 0 (C) -1 (D) None of these

d) $\frac{d\left(\sqrt{x}\right)}{dx}$ = _____

(A) $\frac{1}{2\sqrt{x}}$ (B) $\frac{1}{\sqrt{x}}$ (C) $-\frac{1}{x^2}$ (D) $2\sqrt{x}$

e) $\frac{d\left(a^x\right)}{dx}$ = _____

(A) $a^x \log_e a$ (B) $\log_e a$ (C) $x^a \log_e a$ (D) $a^x \log_e x$

f) $\frac{d(\cos x)}{dx}$ = _____

(A) $-\sec x$ (B) $\sec x$ (C) $-\sin x$ (D) $\sin x$

g) $\frac{d\left(\cot^{-1} x\right)}{dx}$ = _____

(A) $\frac{-1}{\sqrt{1-x^2}}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $\frac{1}{1+x^2}$ (D) $\frac{-1}{1+x^2}$

h) $\int 1\,dx$ = _____

(A) 0 (B) 1 (C) $x+c$ (D) None of these

i) $\int \tan x \, dx =$ _____

(A) $\log \cos ecx + c$ (B) $\log \sec x + c$ (C) $\log|\cos ecx - \cot x| + c$

(D) $\log|\sec x + \tan x| + c$

j) $\int_1^3 \frac{2x}{1+x^2} dx =$ _____

(A) $\log 5$ (B) $\log 2$ (C) $\log 10$ (D) $\log 20$

k) If $\boldsymbol{x}$ = (1, 1, 1) and $\boldsymbol{y}$ = (1, 0, 0) then $\boldsymbol{x} - \boldsymbol{y}$ = _____.

(A) $(1,0,1)$ (B) $(0,1,1)$ (C) $(1,1,0)$ (D) $(0,0,1)$

l) $|2i + j - 3k| =$ _____.

(A) $\sqrt{14}$ (B) $\sqrt{13}$ (C) $\sqrt{41}$ (D) 4

m) If $\bar{x} = 2i - j + k$ and $\bar{y} = i + 2j$ then $\bar{x} \cdot \bar{y} =$ _____.

(A) 1 (B) 0 (C) -1 (D) -2

n) $(2,-1,3) \times (-4,2,-6) =$ _____.

(A) $(-8,-2,-18)$ (B) $(8,2,18)$ (C) $(0,0,0)$ (D) None of these

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) If $\bar{a} = 2i + j - k$, $\bar{b} = i - j + 2k$ and $\bar{c} = i - 2j + k$ then find the direction cosines of $\bar{a} + \bar{b} - 2\bar{c}$. **(5)**

b) If $\bar{a} = i + j + k$ and $\bar{b} = 2i - 2j + k$ then find unit vector perpendicular to $\bar{a}$ and $\bar{b}$. **(5)**

c) Evaluate: $\lim_{x \to 0} \frac{3^{2x} - 2^{2x}}{x}$ **(4)**

Q-3 **Attempt all questions** **(14)**

a) Evaluate: $\lim_{x \to 3} \frac{\sqrt{x+2} - \sqrt{5}}{\sqrt{x+4} - \sqrt{7}}$ **(5)**

b) Evaluate: $\lim_{x \to \frac{\pi}{4}} \frac{2 - \sec^2 x}{1 - \tan x}$ **(5)**

c) Find $\frac{dy}{dx}$ if $y = e^x \sin x \cos x$. **(4)**

Q-4 **Attempt all questions** **(14)**

a) $S = t^3 - 6t^2 + 8t - 4$ gives the distance travelled by a body in t seconds. Find velocity and acceleration at t = 4 second. **(5)**

b) Find $\frac{dy}{dx}$ if $y = \log \sqrt{\frac{a+x}{a-x}}$. **(5)**

c) If $\bar{x} = (-4, 9, 6)$, $\bar{y} = (0, 7, 10)$, $\bar{z} = (-1, 6, 6)$ then show that $(\bar{x} - \bar{z}) \cdot (\bar{y} - \bar{z}) = 0$ **(4)**

Q-5 **Attempt all questions** **(14)**

a) Prove that angle between two vectors $i + j - k$ and $2i - 2j + k$ is $sin^{-1}\left(\frac{\sqrt{26}}{3\sqrt{3}}\right)$. **(5)**

b) Find $\frac{dy}{dx}$ if $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$. **(5)**

c) Evaluate: $\lim_{x \to 2} \frac{x^7 - 128}{x^4 - 16}$ **(4)**

Q-6 **Attempt all questions** **(14)**

a) Forces $(1, 2, 3)$, $(-1, 2, 3)$ and $(-1, 2, -3)$ act on a particles and the particle moves from the point $(0, 1, -2)$ to $(-1, 3, 2)$. Find the work done by the forces. **(5)**

b) Prove that $\int_0^{\frac{\pi}{2}} \frac{\tan x}{\tan x + \cot x} dx = \frac{\pi}{4}$. **(5)**

c) Find derivative of $y = x^3 - 2x$ using first principle. **(4)**

Q-7 **Attempt all questions** **(14)**

a) Evaluate: $\int \frac{(1-3x)^2}{x^3} dx$ **(5)**

b) Find the area of circle $x^2 + y^2 = r^2$. **(5)**

c) For what value of *R*, vectors $2i - 3j + 5k$ and $Ri - 6j - 8k$ are perpendicular to each other? **(4)**

Q-8 **Attempt all questions** **(14)**

a) If $f'(x) = 4x^2 + 6x - 3$ and $f(1) = 2$ then find function $f(x)$. **(5)**

b) If $y = A\cos pt + B\sin pt$ then prove that $\frac{d^2y}{dt^2} + p^2 y = 0$. **(5)**

c) Evalute: $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$ **(4)**